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SPACE 345

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ROYAL AIRCRAFT ESTABLISHMENT

ANALYTICAL GENERATION OF SATELLITE ORBITS AT THE RAE

by

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October 1984

Procurement Executive, Ministry of Defence

Farnborough, Hants

ROYAL AIRCRAFT ESTABLISHMENT

Technical Memorandum Space 345

Received for printing 3 October 1984

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SUMMARY

This Paper traces the development, at the RAE, of computer programs for orbit determination and orbit evolution. The principal program, PROP, is still in regular use, having an accuracy of 50 to 100 m for the orbits of close satellites determined over a period of a few days even when the drag is severe. The chief limitation of PROP is the absence of lunisolar perturbations, but these are included in a companion program, PROD, for the evolution of mean elements.

Work of recent years has focused on a framework for far more accurate representation and generation of orbits. The method is purely analytical (as in PROP) for short-period perturbations, applied compactly via the use of a special system of spherical coordinates, but employs a degree of numerical integration in the variation of mean elements. The goal is a single program that would eventually supersede the PROP-PROD combination, providing an accuracy of perhaps 1 m.

This Paper was presented (5 July 1984) at the 25th COSPAR Meeting, held in Graz (Austria). It will be published in the COSPAR Proceedings (Adv. Space Res., Vol 4). The only change from the presented paper is the footnote on page 3.

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1   EARLY WORK

The Royal Aircraft Establishment became involved in satellite orbit determination within a few days of the launch of Sputnik 1 on 4 October 1957. Kinetheodolites at a number of RAE trials-ranges were adapted to track the satellite*, and a Pegasus computer program was hurriedly produced for the analysis [1] of observations from a single transit. Though kinetheodolites observe azimuth and elevation, the program was written [2] to work with direction cosines (to which conversion is trivial), as the original objective was the processing of data from a radio interferometer specially built at Lasham.

The program was inevitably somewhat primitive and determined only four orbital parameters, quantities equivalent to the standard elliptic parameters i, Ω, ω and M but defined in relation to a crossing (by the satellite) of the latitude of the observing station. This restriction was necessary for program stability, and was convenient in practice since the remaining two parameters, equivalent to a and e, could be obtained from visual observation, over a number of transits, of the orbital period, together with an assumption about the slow variation of the perigee radius.

Results of geophysical significance were obtained when the Pegasus program was used to analyse kinetheodolite observations of Sputnik 2, results that were first recorded in Refs 1 and 2. First, the (secular) variation in Ω, over a period of $4\frac{1}{2}$ months, was 0.7% less than had been predicted; this result, which was controversial at the time, was obtained by fitting a value of $\dot{\Omega}$ to all the individual values of Ω derived, and it amounted (in today's terminology) to a first equation of condition on the even zonal harmonics of the Earth's gravitational field. Secondly, an unexpected secular variation in i was detected, amounting to a daily decrease of about 10^{-3} deg/d; this variation was later interpreted as due to the rotation of the atmosphere and its detection was the starting point for the subsequent RAE discovery that the rotation is (at least for the lower 'upper atmosphere') actually a super-rotation. These and other geophysical results from the early satellites have recently been reviewed by King-Hele [3].

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* The author was then working at RAE Orfordness, where the majority of the kinetheodolite tracking was undertaken, and took part in the first tracking of Sputnik 2 at about 5 am on 9 November 1957 (six days after launch). He was responsible for supervising the film reading and forwarding the data (angles of azimuth and elevation) to the parent establishment at Farnborough. Early in 1958 he transferred to Farnborough (GW Department) and became responsible for the preliminary processing of much of the data from Orfordness and the other RAE outstations, for use in Merson's original computer program.

2 THE PEGASUS PROGRAM BASED ON MERSON'S PERTURBATION THEORY

Though striking successes were achieved by the program that has just been described, perturbations were entirely neglected. This was permissible because each orbit determination used observations from only a very short arc of the orbit; but by the same token, the kinetheodolite observations were not really being exploited to their full accuracy - the accuracy of the derived orbital parameters was equivalent to a positional error of between 1 and 10 km. Thus the inherent limitations of the early program were too deep-rooted for more than interim use - ie, until such time as a proper perturbation theory could be developed, and then implemented on the Pegasus computer.

An appropriate theory was developed by Merson [4] in 1960 and implemented during 1961 by Merson, Gooding and Tayler [5,6].

The orbital elements in this theory are neither osculating nor mean (in the commonly understood sense) but 'smoothed', the significance of this being that first-order short-period perturbations due to J_2 can be fully represented by a variation of smoothed elements that is greatly reduced in comparison with the variation of osculating elements - it entirely vanishes for circular orbits. Because the short-period variation is not entirely eliminated, as it is when mean elements are used, it was decided that smoothed elements should only be published at ascending nodes of the orbit; thus 'time at a node', rather than 'mean anomaly', is the sixth orbital element (the others being a, e, i, Ω and ω) in the computer program, and elements were determined at regular nodal intervals, usually every 25th or every 50th, rather than at epochs uniformly separated in time. The theory ignores short-period perturbations due to J_l , with $l > 2$, but Ref 4 gives explicit formulae for the secular and long-period perturbations due to J_3, J_4, J_5 and J_6 ; corresponding formulae are given for the second-order effects due to J_2 , both in osculating and smoothed elements - the J_2^2 effect on smoothed e was wrongly given as zero in Ref 4 and this was corrected in a subsequent RAE version of the paper (Technical Note Space 26). In implementing the theory, however, no distinction was made between secular and long-period perturbations: both types of perturbations had to be represented via the coefficients of (optional) polynomials (of degree up to 3) that were provided for e, i, Ω, ω and the mean motion n ; the 'constants' in these polynomial were (except for n) the values of the smoothed elements at the epochal node, and the coefficients could be either determined by the program's differential-correction component or held fixed at predetermined values. In practice, first-degree coefficients for e, i, Ω and ω were normally held fixed at values derived from the prior running [6] of separate programs written for the purpose (one to give the contributions to these coefficients due to J_2^2 and J_l , one to cover lunisolar effects, and one to cover certain effects of atmospheric drag

and rotation); occasionally, second-degree coefficients would be used in the same way. Since Merson's theory dealt only with effects due to the Earth's gravitational field, lunisolar effects were based on formulae given by Cook [7] and Gooding [8]. Drag effects were represented in combination between the main program and the separate program: in the main program, the along-track acceleration could be represented empirically by the coefficients of the polynomial for n , whilst the variation in e would be constrained (apart from any polynomial effect) to make $a(1 - e)$ constant; the separate program gave polynomial-coefficient values for the variation of e and ω - also i and Ω to cover atmospheric rotation - proportional to the n variation on the basis of an assumed value for density scale height.

The program described was used for orbit determination for a number of satellites between 1962 and 1968, a good example being for Ariel 2 (1964-15A) [9]. Its main limitation was that only observations expressible as right ascension and declination could be accepted: this allowed other forms of directional measurement, in particular azimuth and elevation or direction cosines, since these could be transformed to right ascension and declination, but the inability to accept range data was a serious defect; there was no possibility of range rate (Doppler) measurements being used, as the program did not even compute the velocity of the satellite, the theory only being taken as far as the generation of position. The accuracy was much superior to that of the earlier program, however: typically, orbit determination (and hence the generation of inter-polated satellite position) was accurate to $\frac{1}{2}$ km over a two-day period [9]. For close-earth satellites the limitation on accuracy was always the unpredictability of drag, and it was this - not the failure to distinguish between secular and long-period perturba-tions - that restricted the applicability to periods sometimes as short as two days. The inadequacy of the long-period representation over longer periods, as well as the unsatisfactory business of having to run separate programs before the main programs, was soon recognized, however, and it was a prime candidate for improvement when (as we see in the next section) it became inevitable that a new program would have to be written.

Two concluding points are worth making about Merson's theory, as implemented in the Pegasus computer program. The first is that, though the J_2 theory was, in itself, free of singularity, sources of singularity were present in the program: thus, 'time at a node' is an unsatisfactory element for an equatorial orbit, where nodes are undefined, and the same is true of the elements Ω and ω ; for Ω and ω the only effect is an ill conditioning of partial-derivative matrices, but failure in nodal-time definition is more serious, as this undermines epoch itself. Singularities could also occur in the programs run before the main program, since infinities occur in the rate of change of Ω for equatorial orbits and in the rate of change of ω for circular and/or equatorial orbits.

The second point is that the Pegasus elements were in certain respects superior to the elements used in the replacement program. As remarked in Appendix C of Ref 10,

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the residual short-period variation of the smoothed elements, which was a nuisance, could
have been dealt with by defining a set of 'mean smoothed elements', including M, to
coincide with smoothed elements at ascending nodes. They would then qualify as 'mean
elements' in the definitive sense of a recent general study [11] and it is possible to
state the values of the k-constants, in equations (138)-(146) of Ref 11, that are
associated; thus k_a, k_e, k_i, k_Ω, k_ω and k_M are given by

$$k_a = \frac{1}{3}h(2 + 3e^2) + \frac{3}{4} f e^2 \cos 2\omega \ ,$$

$$k_e = \frac{4}{3}h(3 + 2e^2) + 3f \cos 2\omega \ ,$$

$$k_i = 1 + \frac{4}{3} e \cos \omega \ ,$$

$$k_\Omega = 2(v - M)_{u=0} - \frac{8}{3} e \sin \omega \ ,$$

$$k_\omega = -16(v - M)_{u=0} + \frac{1}{3}\left[32 \ eg \sin \omega - (9f + 4e^2h) \ \sin 2\omega \right]$$

and $\quad k_M = -3f \sin 2\omega \ ;$

here $f = \sin^2 i$, $h = 1 - \frac{3}{2}f$ and $g = 1 - \frac{5}{4}f$, whilst the suffix in $(v - M)_{u=0}$
indicates an evaluation at the ascending node, u being the argument of latitude
$(= \omega + v)$. If mean smoothed elements had been defined to coincide with smoothed
elements at descending nodes, rather than ascending nodes, the signs of the terms in
$\sin \omega$ and $\cos \omega$ would be reversed and the suffix on $(v - M)$ would be '$u=\pi$'.
It then follows that, if the elements had been defined by 'splitting the difference'
between ascending and descending nodal bases, that (in particular) k_i would be unity
and mean inclination would have coincided with the most natural value recommended in
Ref 11; k_a is in any vase the recommended value (postscript to Ref 11), so two
advantages of the Pegasus elements (over the subsequent PROP elements) have been
demonstrated. A full comparison of 'Pegasus elements' with those introduced by Kozai [12]
(and hence underlying PROP) was made in Ref 13.

3   THE FORTRAN PROGRAM PROP

There were three overriding reasons for wishing to replace the Pegasus program.
First, being written almost entirely in Pegasus machine orders, it could not survive the
eventual demise of Pegasus itself. Second, it was inconvenient to continue using (mean
smoothed) orbital elements that were not recognized outside RAE. The SAO (Smithsonian
Astrophysical Observatory) were using a computer program known as [14] DOI-3 (for
differential orbit improvement), based on Kozai's theory [12], and ESRO (European Space
Research Organisation) were in the process of adopting [15] the same theory for their
DCOP [16] (differential correction orbit programme). When a new RAE program, in

Standard Fortran [17] for maximum portability, was planned, therefore, in 1965, it seemed right to base the underlying theory (for short-period perturbations due to J_2) on the SAO/Kozai mean orbital elements; with hindsight, however, this was a retrograde step in relation to at least two of the orbital elements (a and i), as already mentioned - the work described in section 5 effectively involves a return to Merson's a and i . The third overriding reason for replacing the Pegasus program related to its main limitation in use, viz the restriction to angular observations.

The proposed program was given the name PROP (program for the refinement of orbital parameters) and its dynamic model was developed and described by Merson [18]. A users' manual [10] was in due course written for the PROP3 version of the program and an update generated for the PROP6 version [19,20]. The program has been extensively used both inside and outside the RAE - it has been estimated that some 10,000 individual orbit determinations have been carried out - and an exhaustive documentation is available [21] for the version of the program used at the University of Aston in Birmingham.

An important feature of the underlying theory, as described in Ref 18, is that (like the Pegasus program) it is entirely free of singularity, even though the normal elliptic elements (a, e, i, Ω, ω and M) are used. This was achieved by arranging that both short-period and long-period perturbations would be applied in composite manner, the six perturbations actually computed being δi, $\delta\Omega \sin i$, $\delta u + \delta\Omega \cos i$, $\delta p/2p$, $\delta r/p$ and $\delta \ddot{r}$, p being the parameter (semi-latus rectum) given by $p = a(1 - e^2)$. This was a significant improvement on Kozai's approach, since the only composition of perturbations in Ref 12 is of δa, δe, $\delta\omega$ and δM into δr and δu, and the long-period δu is singular for equatorial orbits; further, the retention of six perturbed quantities in PROP permits velocity to be covered as well as position. As the program developed, however, it was found [20] desirable to replace the last three composite perturbations (long-period only) by δa, δe and $e \delta v$. (An altogether more natural approach to the composition of short-period perturbations has been developed in the last few years, as part of the post-PROP studies described in section 5.)

As with the Pegasus program, short-period perturbations are present in PROP just for the representation of first-order J_2 effects, and this immediately limits the accuracy, for close-earth satellites, to around 50 m. Secular and long-period perturbations are present for the representation of J_2^2 effects and (first-order) J_l effects for l ranging from 3 to as high as it is desired to go - the current PROP6 version of the program would have to be recompiled if it were desired that l exceed 40. The first-order (secular and long-period) effects of J_l are represented completely - see Ref 11 for a discussion of 'order' in this context - and this means, in particular, that the representation is correct for orbits that are near-circular [22] or near-equatorial or both. The representation is via recurrence relations [8,18], and these are very efficiently implemented - the appropriate quantities (functions of the epoch elements a, e, i and ω) are built up at the beginning of each iteration of the differential-correction process and there is no unnecessary recomputing.

The correct representation of long-period perturbations is an important feature of PROP, but the most important new feature, in relation to the preceding Pegasus program, has been, as already remarked, the ability to handle observations other than angular: thus the program's first application was to range and range-rate observations of GEO 2 (1965-81A) [23]. The facility to process no less than 16 types of observation was built into the original version of the program; for example, Types 1, 2, 3, 4 and 7 refer, respectively, to observations of ρ (range) alone, α and δ (right ascension and declination), l and m (direction cosines, of the sort covered by the previous programs),

alone, and simultaneous observations of ρ, α and δ . Observations of azimuth and elevation are not processed separately but, as with the Pegasus program, converted to α and δ at the input stage, corrections for refraction etc being made at the same time, as appropriate. Input, which is part of PROP and not ancillary as with the Pegasus program, is associated with the sources of the observations (ie the organizations concerned) rather than with the observation types, so that their observation formats can be used directly. Some of the effects of this dissociation are: that there are several sources of, for example, Type 2 observations, all distinction having been lost by the start of the differential-correction process; that for some of the types of observation, eg Type 5 ($\dot{\alpha}$ and $\dot{\delta}$), there have so far been no sources of observations, so that these types have been redundant; but that new sources of data can easily be accommodated (with activation of a redundant type if necessary).

The basic orbital elements of PROP are, as already intimated, the Kozai mean elements e, i, Ω, ω and M , the element a being defined from n (= $\dot{M}$) by a J_2-modified version of Kepler's third law. It is important to realize that e (similarly i , etc) differs from osculating e only in respect of the short-period perturbation; the use of a 'mean mean' element, obtained by removal of a nominal long-period perturbation from the mean element, was eschewed from the start so that the program would be free from any singularity in the vicinity of the critical inclinations. Another way of saying this is that long-period perturbations in PROP are represented by definite integrals, the lower limit of the integration being the epoch at which the elements are being determined. Epochs in PROP are constrained to be midnights, but it would be easy to remove this restriction.

The mean elements, e etc, are allowed to vary in two ways as t departs from its epoch value, t_o . First, explicit polynomial variation (of degree up to 5, with the variation of M inducing a derived variation in n) is allowed, the polynomial coefficients being denoted by e_j etc (j = 0, ... , 5) . Thus e_o, i_o, Ω_o, ω_o and M_o are the values of the mean elements at epoch; M_1 is an obligatory sixth element equal to n_o and thereby defining a_o); M_2, ... , M_5 are optional elements that (in practice) permit an empirical representation of the along-track acceleration due to atmospheric drag; components of e_j and i_j are determined from n_j (= (j + 1) M_{j+1}) to represent residual atmospheric effects; and components of Ω_1 and ω_1 are determined from the secular effects of the even J_l (including the J_2^2 effect). Residual components of e_j, i_j, Ω_j and ω_j , known [10] as 'exclusive elements', are permitted, for use in modelling perturbations that are not explicitly covered by PROP, but they are

almost never used in practice. The other way in which the mean elements are allowed to vary is by incorporation of the long-period perturbations, these being defined (as already made clear) so as to be zero at epoch; this variation is implicit, rather than explicit, however, as the perturbations are applied in the non-singular manner that has been described. (A source of confusion in notation arises here: mean elements are often [11] denoted by $\bar{e}$ etc, PROP's e_o being the Kozai $\bar{e}$ at epoch; since the long-period variation in PROP elements is not explicitly added to the polynomial variation, however, Refs 18 and 10 reserved the notation $\bar{e}$ for the polynomial variation - the notation has been avoided here, to prevent misunderstanding.)

A complete summary of the orbital model of PROP is given in Appendix B of Ref 24, the users' manual for PREP (program for realizing ephemeris print-outs), which is the program for ephemeris generation paralleling PROP for orbit determination. Other programs based on the PROP model are referenced in Ref 20.

For each basic element the set of k selected elements in the model (polynomial coefficients e_j , with $j = 0, \ldots , k-1$) are divided into two subsets: the first m coefficients ($0 \leqslant m \leqslant k$, so either subset can be empty) are parameters of the differential-correction process, whilst the remaining (k - m) are held fixed. Partial derivatives of all the observations are computed with respect to the parameters (elements of the first subset) during each iteration of the process. The computation is purely analytical, allowance being made for first-order secular variation due to J_2 , but for no other source of perturbation. At two critical points of the process (during each iteration), PROP can, at the user's option, modify the standard set of elliptic elements so as to avoid convergence problems associated with near-circular or near-equatorial orbits. Full details are given in Ref 10; for an orbit with e close to zero, one modification, which avoids an ill-conditioned derivative matrix, effectively involves the replacement of partial derivatives wrt ω_o and M_o by derivatives wrt ω_o and U_o ($= M_o + \omega_o$), whilst another, which avoids the necessity for non-linear parameter correction, involves the temporary introduction of elements $e_o \cos \omega_o$ and $e_o \sin \omega_o$ to replace e_o and ω_o . The former modification relates to the high negative correlation between ω_o and M_o when e_o is small; the PROP user should also respond to this situation when tabulating orbital elements obtained by the program, since the proper procedure is to quote ω_o only to the number of figures that are actually meaningful, while quoting the composite parameter ($M_o + \omega_o$) to more decimal places than would be appropriate for M_o .

PROP is still in regular use. Its main limitation is probably the total absence of any explicit representation of lunisolar perturbations, though some implicit allowance can be made, as in the Pegasus program, via the 'exclusive' polynomial coefficients that have been described. For close-earth satellites, over a period as short as one day, the error is no more serious (say up to 50 m) than that due to the neglect of short-period perturbations for J_l with $l > 2$, but the effect rapidly becomes more serious for more distant orbits.

The only tesseral (or rather sectorial) harmonic represented is $J_{2,2}$. Even with the Pegasus program it was realized, during the Ariel 2 analysis [9], that the dominant

g-track effect of the $J_{2,2}$ perturbation was important enough to be added to the
tal model, and this effect was built into the earliest version of PROP. In due course
as decided that the dominant (semi-diurnal) $J_{2,2}$ effect in all the elements should
epresented, and Ref 20 gives the details.

The overall accuracy of which PROP is capable is of the order of 50 m. The accuracy
distribution of observations are not always good enough to achieve this, of course,
for Ariel 4 (1979-109A), which was a good example of the definitive use of PROP for
.t determination to be associated with the experimenters' telemetry analysis, an
iracy of 120 m was quoted [25], the main source of error being the Minitrack data.

A final point to mention, concerning PROP in relation to the preceding Pegasus
gram, is that a change was made in the choice of equinox to define the x-axis.
igh both programs took the equator of date as xy-plane, the apparently natural choice
equinox of date, made for the Pegasus program, was abandoned for PROP in favour of a
si-equinox defined by projection of the mean equinox of 1950.0. The basis for this
rid system is given in Appendix E of Ref 20; it follows Kozai and the SAO.

PROD. A COMPANION PROGRAM TO PROP

The function of PROP is to refine a set of Kozai mean orbital elements, and usually
e other parameters to represent drag effects, by fitting to observations of a satellite
interest, collected over a number of days. Over such a short period the absence of
isolar perturbations is not serious; but when it comes to interpreting sets of
ments (usually at uniform intervals of time) spanning a period of perhaps several
ths, the neglect of these perturbations can in most cases no longer be tolerated.
t was wanted, therefore, as a companion to PROP, was a program that would generate
evolution of a set of mean elements over long periods, allowing for lunisolar
vitational attraction as well as for the zonal harmonics of the Earth. (Except for
its experiencing resonance, referred to again at the end of this section, or when
reme accuracy is required, it is legitimate to neglect the tesseral harmonics, since
general they cause perturbations that have periods of less than a day.) PROD
ogram for orbit development) was developed with this rationale.

The program operates by numerical integration of Lagrange's planetary equations,
'ariation of parameters' method. The method of integration is a fourth-order
ge-Kutta process, which is not very efficient over long periods of time. No
ficulty has been found in practice, however, mainly because a fairly long integration
p time can be used since no short-period terms are carried in Lagrange's equations.

A full account of the theory underlying PROD was given by Cook [26], and the
gram has been regularly used for assessing the lifetimes (via a determination of the
igee-height variation) of newly-launched satellites in highly-eccentric orbits. The
n features of the theory and program are now outlined together.

The program only covers the elements a, e, i, Ω and ω. The sixth element (M)
included in the theory, and it was intended to add M to the program, but in practice

the evolution of M has never been required - since tesseral-harmonic perturbations are not represented, the rates of change of the mean elements a, e, i, Ω and ω are independent of M. (Further, the variation in a is always set to zero.)

Only gravitational perturbations are represented, though the original intention was to cover drag and solar radiation pressure as well. The Earth's zonal harmonics are covered essentially as in PROP, with first-order secular and long-period perturbations represented up to J_{10}, as required, together with second-order J_2 perturbations - there is an error in the J_2^2 rate of change of i, as quoted in Ref 26, since this rate of change is proportional to $\sin 2\omega$ not $\cos 2\omega$. The contributions of the J_1 to the rates of change are generated by recurrence relations, similar to those in PROP though computed somewhat less efficiently. Since the standard elliptic elements are integrated directly, there is no avoidance of singularity as there is in PROP and this is probably the main defect of the program. (A distinct program, known as PROD2, was developed to cope with near-circular and near-equatorial orbits, but this is too slow for general use as the Fortran coding makes liberal use of complex variables.)

Lunisolar perturbations are represented in such a way that the user, via four digits of one of the quantities on the control card (part of the data input), has a pair of options for both the lunar and the solar components of the element rates of change. The first option relates to the number of harmonic terms used in the standard expansion of the disturbing function due to the attracting body - see equation (10) of Ref 26 or equation (22) of Ref 8. A single term should always be adequate for the solar attraction, in view of the Sun's great distance, but at least two will normally be required for the lunar attraction (the second, involving a harmonic function of degree 3, being known as the parallactic term) and sometimes (for satellites in highly eccentric orbits) it may be necessary to take as many as four - the limit allowed is eight, ie harmonics up to degree 9. The other option permits a simple choice between a singly averaged disturbing function, ie averaged only with respect to the mean anomaly of the satellite (to avoid short-period perturbations), and a doubly averaged disturbing function, ie averaged also with respect to the mean anomaly of the attracting body itself. It is important to realize that it is not just a matter of excluding certain terms when double averaging is opted for, since there is a fundamental difference in the way the terms of the disturbing function are developed - with single averaging the attracting body is effectively at an arbitrary fixed point in space (to be specified by a subroutine) at each integration point, whereas with double averaging it is only the orbit of the body that is relevant, the body being effectively replaced by a ring of mass distributed around that orbit. Far from requiring fewer terms, this means that double averaging actually requires more; thus equations (59)-(63) of Ref 20 involve triple summation, whereas equations (31)-(35) involve only double summation. In practice, because the Earth-Sun orbital period is so long (as well as because fewer terms are involved!), it will be normal to take the solar function with only single averaging; for the lunar function, on the other hand, it will often make sense to opt for double averaging, since it would otherwise be necessary to restrict the integration step time to being a very small fraction of a month.

The most serious limitation of PROD is the absence of all tesseral-harmonic representation, which, together with the omission of sixth-element variation, precludes the coverage of resonance effects. Such effects can, of course, be of critical importance, in particular for 24h and 12h orbits. It is worth mentioning, therefore, another program that is unrelated to either PROP or PROD. Known as TCSKEF, it originated as a program used by the Royal Air Force for the quick generation of ephemerides for Skynet communications satellites, but was taken over by RAE and made available as a general-purpose tool for predicting mean orbital elements, ephemerides and look angles. The orbit generator has been documented elsewhere [27] and it suffices here: first, to remark that, though not totally analytic like PROP, TCSKEF is much more analytic than PROD, in that it is finite increments to elements, rather than their rates of change, for which formulae are available; and secondly to summarize the sources of perturbation covered. Only one zonal harmonic, J_2, is included, and this of course is an immediate limitation on applicability; as described in Ref 27, the main 24h resonance effects are covered via the tesseral harmonics $J_{2,2}$, $J_{3,3}$ and $J_{3,1}$, and the main 12h effects via $J_{3,2}$, but the orbit generator has since been enhanced so that some additional effects of resonance are explicitly covered and others can easily be added; lunisolar attraction is represented, but only for the leading harmonic term in the expansion of the disturbing function, by a procedure that gives an appropriate weighting to the effects associated with single and double averaging; a primitive representation of the effects of eclipse-free solar radiation is included.

5 RECENT DEVELOPMENTS

After PROP and PROD, work at RAE concentrated on orbit generation by Cowell's method, also known rather misleadingly as 'special perturbations', which is at the opposite end of the spectrum from the 'general perturbations' method underlying PROP, since it is based on pure (and highly efficient) numerical integration - rates of change of position and velocity coordinates are integrated, so the method does not recognize such a concept as 'orbital element'. Cowell's method is employed for its superior accuracy, and the pair of programs used with the Skynet 2 communications satellites were, for orbit determination and ephemeris generation respectively, SPOD2 [28] and SKEPH2 [29]. Generalized versions of these programs, permitting, eg, the use of arbitrarily extensive geopotential models, have been developed since then; known as POLO and PINTO respectively, they have not yet been documented. No more will be said about any of the programs, however, as their purely numerical nature puts them outside the scope of the present paper.

It occurred to the author early in 1980, after completion of the orbit theory that was in due course published as Ref 11, that a new approach to orbit generation (and eventually orbit determination) was possible, which would combine the speed and sophistication of analytical generation with the accuracy of pure numerical integration. A mixed analytical-numerical procedure would thereby be adopted that could be described as semi-analytical, though (just as with the program TCSKEF, referred to at the end of the last section), the numerical element of the procedure would be relatively slight, amounting merely to the splitting of any excessively long period of time, over which mean elements are to be propagated, into subperiods.

There is nothing intrinsically original in the idea of a mixed procedure. The novelty of the approach lies in the particular interface c. .ren between the analytical and numerical components of the orbit generator. Two aspects are of fundamental importance here. The first concerns the treatment of short-period perturbations, which is purely analytic and involves their expression in a (geocentric) system of spherical-polar coordinates based on an optimally defined mean orbital plane. (Ref 11 was based on cylindrical coordinates, but when the complete set of formulae for J_2^2 perturbations were developed [30] it was realized that spherical coordinates were better.) The second aspect of the interface concerns mean elements, which are conceptually defined so as to be free of terms involving the satellite's mean anomaly (the normally defined short-period perturbations) and of terms involving the Greenwich sidereal angle (in perturbations due to the tesseral harmonics), except inasmuch as these quantities appear together in terms that must be regarded as resonant - an arbitrary criterion for resonance is required here. The precise definition of the mean elements involves the assignment of arbitrary constants in the analytical integration (the k-constants of section 2 and Ref 11, as far as J_2 perturbations are concerned), and there are significant rewards (in the simplicity of the expressions for short-period perturbations) if the right assignment is made - the assignment for mean i and Ω effectively defines the mean orbital plane.

Results from a preliminary test of the new approach were reported [31] at the 1980 COSPAR meeting in Budapest. The test program covered the potential field associated with J_2 and J_3 (as specimen zonal harmonics), $J_{2,2}$ (as a prototype tesseral harmonic), the Sun and the Moon. Short-period perturbations (in cylindrical coordinates) were restricted to e-independent terms and the program was not singularity-free for equatorial orbits. The results were promising, however, the main source of error, for an orbit of period 4 h, eccentricity 0.002 and inclination 70°, being an unmodelled coupling between J_2 and $J_{2,2}$ that led to an along-track error signature of period 6 h and amplitude about 0.7 m; the errors were much greater for a higher orbit (18h period for the test case), the inadequately modelled factor then being the motion of the Moon.

The next stage of the work was to complete the analysis of short-period perturbations of second order in J_2 and first order in J_3, to make them valid for (elliptic) orbits of arbitrary eccentricity, at the same time removing the source of singularity. The results, now most compactly expressed in spherical coordinates, were presented [30] at the 1982 IAF meeting in Paris; for an orbit of period 4 h, eccentricity 0.5 and inclination 40°, with only J_2 and J_3 represented, the dominant error (against an integrated yardstick ephemeris) was found to be in apogee position, growing at about 6 cm per revolution and due to coupling between J_2 and J_3. A full account of the underlying analysis is to be issued shortly [32] and work is now concentrated on the coverage of $J_{2,2}$ for orbits of arbitrary (< 1) eccentricity.

The eventual goal is a fast program that would replace the PROP/PROD combination and provide an accuracy of order 1 m. Many steps remain to be taken before this goal can be seen as realistic, and some of these are listed as a conclusion to this paper.

(a)   The first order coverage of J_3 must be extended to an arbitrary zonal harmonic; implementation of this will only be straightforward if recurrence relations can be found for the perturbations in spherical coordinates.

(b)   The present $J_{2,2}$ study must be brought to a satisfactory conclusion, and then extended to arbitrary tesseral harmonics, again with the use of recurrence relations.

(c)   A sufficiently accurate representation of lunisolar perturbations must be included - in practice it is likely that the epithet 'sufficiently accurate' will be defined on the basis of an upper bound on a and e for the specified accuracy to be achievable.

(d)   Atmospheric drag must be covered, unless the program is to remain unusable for the vast number of orbits below some (perigee) altitude that is itself a function of the satellite's mass/shape characteristics and the solar-cycle variation in atmospheric density.

(e)   Other perturbations, such as those associated with the tides and those due to solar radiation, must be covered, as well as the effects of the Earth's precession and nutation.

(f)   Sufficiently accurate partial derivatives must be programmed, so that the program routinely converges to the right results.

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REPORT DOCUMENTATION PAGE

Overall security classification of this page

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1. DRIC Reference (to be added by DRIC)	2. Originator's Reference RAE TM Space 345	3. Agency Reference N/A	4. Report Security Classification/Marking UNCLASSIFIED

5. DRIC Code for Originator 7673000W	6. Originator (Corporate Author) Name and Location Royal Aircraft Establishment, Farnborough, Hants, UK

5a. Sponsoring Agency's Code N/A	6a. Sponsoring Agency (Contract Authority) Name and Location N/A

7. Title   Analytical generation of satellite orbits at the RAE

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials Gooding, R.H.	9a. Author 2	9b. Authors 3, 4	10. Date October 1984	Pages 16	Refs. 32

11. Contract Number N/A	12. Period N/A	13. Project	14. Other Reference Nos.

15. Distribution statement

    (a) Controlled by —

    (b) Special limitations (if any) —

16. Descriptors (Keywords)   (Descriptors marked * are selected from TEST)

Orbits*.   Artificial satellites*.   Perturbation*.   Least squares*.

17. Abstract

    This Paper traces the development, at the RAE, of computer programs for orbit determination and orbit evolution. The principal program, PROP, is still in regular use, having an accuracy of 50 to 100 m for the orbits of close satellites determined over a period of a few days even when the drag is severe. The chief limitation of PROP is the absence of lunisolar perturbations, but these are included in a companion program, PROD, for the evolution of mean elements.

    Work of recent years has focused on a framework for far more accurate representation and generation or orbits. The method is purely analytical (as in PROP) for short-period perturbations, applied compactly via the use of a special system of spherical coordinates, but employs a degree of numerical integration in the variation of mean elements. The goal is a single program that would eventually supersede the PROP-PROD combination, providing an accuracy of perhaps 1 m.